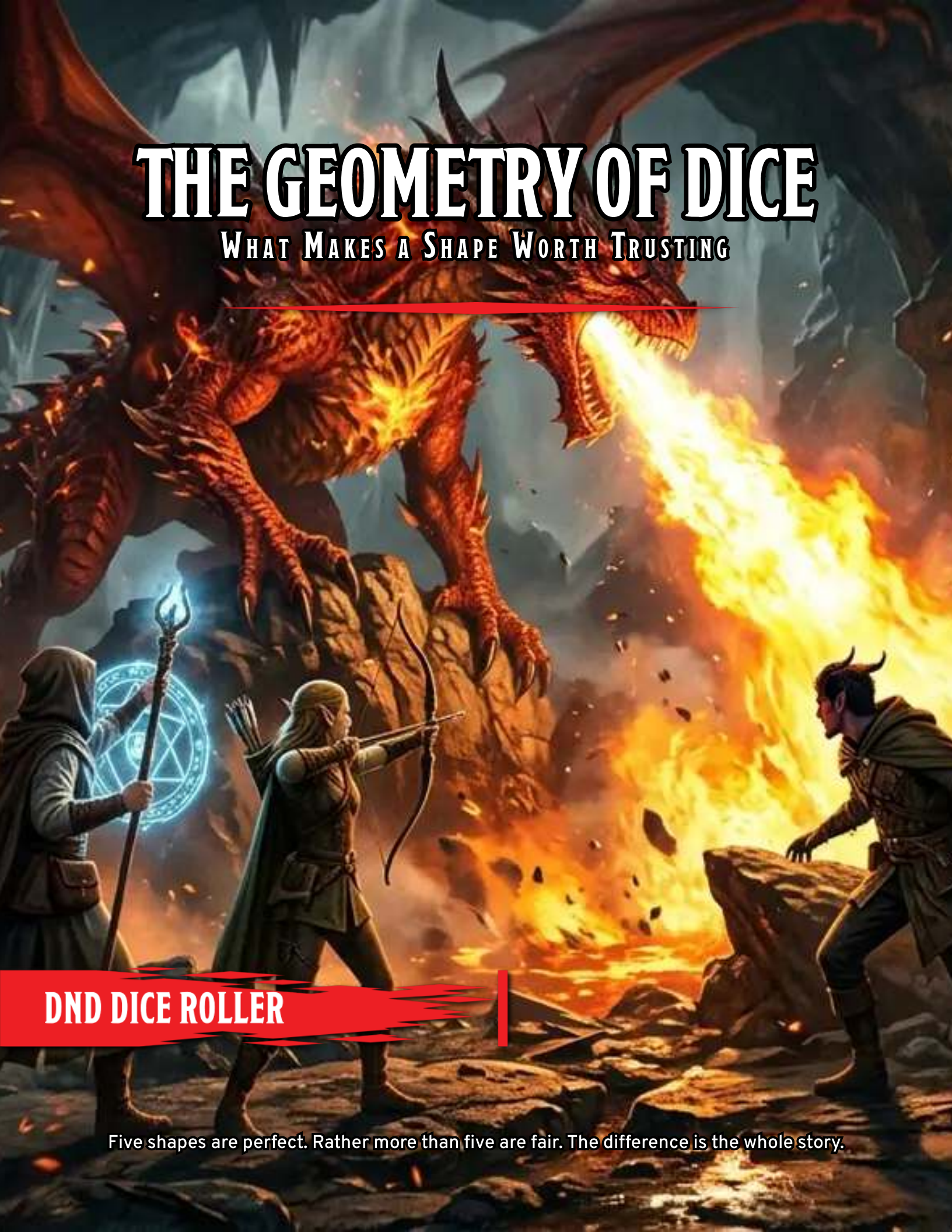
THE GEOMETRY OF DICE
What Makes a Shape Worth Trusting
DND DICE ROLLER
Five shapes are perfect. Rather more than five are fair. The difference is the whole story.

What Makes a Die Fair

The obvious answer is symmetry — and the obvious answer is very nearly right, but not quite. It matters *which* symmetry, and the usual assumption asks for far more of it than a die actually needs.

A die is fair when no face is favoured. For that, every **face** has to be equivalent to every other one: same shape, same size, and sitting in the solid the same way, so the die can be turned to put any face where any other face was.

Notice what that does **not** require. It says nothing about the edges, and nothing about the corners.

The Rule Faces equivalent to each other — that is the whole requirement. A die whose corners are all different can still be perfectly fair, so long as you are reading the faces.

The Five Perfect Solids

Ask instead for *everything* to be equivalent — every face, every edge, every vertex — and you have defined a **regular** polyhedron. Insist it also be convex, with no dents, and there are exactly five in existence.

Five. Not five that happen to be convenient; five that can be. There is a proof that no sixth is possible, and it is older than almost anything you would think to compare it to.

The Platonic Solids

Solid	Faces	Face shape	Die
Tetrahedron	4	triangle	d4
Hexahedron (cube)	6	square	d6
Octahedron	8	triangle	d8
Dodecahedron	12	pentagon	d12
Icosahedron	20	triangle	d20

The mathematics is usually credited to Theaetetus, a generation before Plato. The four elements came from Empedocles before that. Plato's move was to weld the two together.

Icosahedron

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Plato's Likely Story

In *Timaeus*, around 360 BCE, Plato matches four of the solids to the four elements. He is careful about how he says it — the account is offered as the *likely* one, argued from what the shapes are like, not asserted as fact.

The argument runs on sharpness, size and mobility. Fewest faces means the most acute angles, so that solid is the most piercing and the most mobile. Most faces means the roundest, and the most sluggish.

The Four Elements

Element	Solid	Die	Argued from
Fire	tetrahedron	`d4`	fewest faces, sharpest, lightest
Air	octahedron	`d8`	between the others on every count
Water	icosahedron	`d20`	largest and roundest, least mobile
Earth	cube	`d6`	square faces, the most stable base

Why Earth Cannot Burn Underneath the imagery is a geometrical argument. Plato builds the solids from two right triangles: half an equilateral, and half a square. The `d4`, `d8` and `d20` are faced with equilateral triangles and share the same stock, so fire, air and water can be taken apart and rebuilt into one another. The cube's square faces come from the *other* triangle — earth is made of different material, and cannot transform at all.

A pentagon can be built from neither, which is why the `d12` is left out of the four. Plato says only that there was a fifth combination, used for the whole universe, and leaves it there.

Kepler's Beautiful Mistake

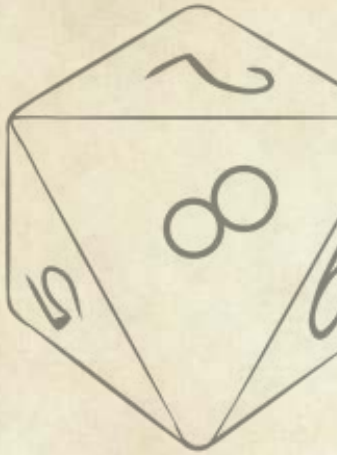
In 1596 Johannes Kepler published *Mysterium Cosmographicum*, proposing that the spacing of the six known planets was set by the five Platonic solids nested one inside another — a cube between Saturn and Jupiter, a tetrahedron between Jupiter and Mars, and so on inward.

Five gaps between six planets, and exactly five regular solids to fill them. Kepler could not believe that was a coincidence. He had no way of knowing there were more planets to come.

It is wrong, and it is one of the loveliest wrong ideas in the history of astronomy. He was still fond of it decades later, long after the elliptical orbits that actually made his name.

The Tetrahedron Problem

The `d4` is the one Platonic solid that makes a poor die: with four faces and sharp points it barely rolls, and it lands with a *point* up rather than a face, so the number has to be read off the bottom edge. Modern versions truncate the tips and put the numbers on the small flats instead.



Tetrahedron & Octahedron

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Building Dice to Order

Face equivalence is a much cheaper requirement than regularity, and once you stop insisting on regular solids you can build a fair die with almost any number of sides you like.

Two pyramids, base to base

Take a pyramid with any number of faces, and glue it to its mirror image. Every face is equivalent to every other; the edges and corners are not. The shape is a **trapezohedron**, and it gives you an even-numbered die of any size you please — `d10`, `d14`, `d16`, and on up to sizes nobody has any use for.

The `d8` is the special case where the result happens to come out regular.

The d10's Little Offset Look closely at a `d10` and the faces do not line up across the middle — they are staggered. They have to be. With five faces on each side of the mirror plane, lining them up would leave an *edge* pointing at the ceiling when the die settles, instead of a face. The stagger is what puts a number on top.

An illusion worth noticing

A `d8` looks as though it has one obvious waist, because the numbers are all printed upright relative to a single plane. It does not. Turn it and you will find three square cross-sections through it, not one. The numbering invents a top and a bottom the shape does not have.

Doubling up for odd numbers

Trapezohedra are always even. The straightforward fix for odd numbers is to repeat yourself: number a `d6` as **1,1,2,2,3,3** and you have a `d3`.

A `d10` numbered in pairs is a `d5`. A `d14` gives a `d7`. Players did this with plain dice for years before anyone bothered to print them — *one or two is a one, three or four is a two* — and the printed versions simply save the arithmetic.

So the d2 Is Real After All

It is a `d6` with each set of three faces rounded together into one continuous curved side, leaving a die with two faces and a rather satisfying roll. You could flip a coin instead. But you came here to roll dice.

Rounded and beveled

The other route to an odd number is to take a bar with a triangular cross-section and round its long edges away, so the die cannot rest on them. Three flat faces, all identical, all fair. Bevel those edges to a ridge instead of rounding them and you get the crystal-shaped dice sold as amethyst shards and rocket ships.

Pentagonal Trapezohedron

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Catalan Solids

Here is where dropping the extra symmetry really pays. Keep every face equivalent, throw away edge and vertex equivalence entirely, and you arrive at a family described by the Belgian mathematician **Eugène Charles Catalan** in 1865.

Most of them are Platonic solids with their faces pushed out into shallow pyramids. Take a cube and raise each square face into a four-sided pyramid: six faces, four sides each, and you have a fair `d24`. Do the same to an icosahedron with three-sided pyramids and you get a `d60`; with six-sided ones, a `d120`.

Some Fair Non-Regular Dice

Die	Solid	Built from
`d24`	tetrakis hexahedron	cube, faces raised
`d48`	disdyakis dodecahedron	cube or octahedron
`d30`	rhombic triacontahedron	—
`d60`	deltoidal hexecontahedron	icosahedron
`d120`	disdyakis triacontahedron	icosahedron

The vertices are plainly not all alike — the peak of a raised pyramid is nothing like a corner of the original cube. It does not matter. You read faces.

Dodecahedron

Where It Stops Working

Two limits, one practical and one real.

Too round to settle

The `d120` is larger than a golf ball and very nearly spherical. It rolls for a long time, and when it stops you have to hunt for the number among a hundred and nineteen others. The `d100` Zocchihedron has the same problem — which is why most tables read two `d10`s instead, one for tens and one for units.

Too irregular to trust

Then there are the dice with no symmetry at all — faces of different shapes and sizes, made by spacing points around a sphere as evenly as possible and slicing flat where each point lands. It is a reasonable way to *approximate* fairness. It is not fairness. Roll one of those enough times and the bias shows up in the tally.

Does It Matter? For a `d7` you are using to pick a random encounter, no. These shapes turn up in games where the referee is improvising half the rules anyway. A mild bias in a novelty die is not the thing that will decide your session.

Roll Any of Them

Every die in this booklet is on the roller at **dnddiceroller.com** — where the shapes are perfectly fair, because they are not shapes at all.

DND DICE ROLLER

Roll with us

dnddiceroller.com is the free roller that remembers your lists, follows you from laptop to phone, and never asks you to install anything. Six thousand of you a day can't be wrong. Well, you can, but the dice aren't.

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